Lesson 9: **Sum of the Interior Angles of a Polygon**

Lesson Topic

Decomposing polygons into triangles to find a formula for the sum of the interior angles of a polygon with *n* sides.

Grades

4–5

Lesson Length

50–75 minutes

NCTM Standards Addressed

- Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes
- Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions
- Use geometric models to solve problems in other areas of mathematics, such as number and measurement
- Understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute
- Explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way
- Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life

PA Standards Addressed

- Give formal definitions of geometric figures.
- Identify properties of geometric figures (e.g., parallel, perpendicular, similar, congruent, symmetrical).

Student Objectives

Students will:

- see how all polygons can be decomposed into triangles.
- discover the sum of the measures of the interior angles of a triangle.
- review names of geometric figures.
- find a rule for determining the number of triangles that a polygon with *n* sides can be decomposed into by drawing diagonals from a single vertex.

- find a rule for finding the sum of the measures of the interior angles of a polygon with *n* sides.
- learn to use the Geometer's Sketchpad™ to explore polygons and make and test conjectures.

Grouping for Instruction

- Whole class for launch and closure
- Small groups of 4–6 for the investigation

Overview of Lesson

Students use the Geometer's Sketchpad™ software to create and dynamically manipulate triangles, measure interior angles, and find their sum. They then repeat the process for quadrilaterals and pentagons. Students make a conjecture and investigate this conjecture using paper and pencil. Students discover a relationship between the number of sides of a polygon and the number of triangles that the polygon can be decomposed into by drawing diagonals from a single vertex. They also discover a relationship between the number of triangles in the decomposition and the sum of the interior angles. Students find a formula for the sum of the interior angles of a polygon and use the formula to solve a problem.

Background Information

Students should be familiar with basic shapes and their definitions and properties. Students should know the meaning of the terms diagonal and vertex. Students should have limited comfort with the concept of a variable. Students must know how to use a mouse.

Materials and Equipment

- For each team
  - Blank paper
  - Rulers
  - Calculators
  - Computer with the Geometer's Sketchpad™ software loaded on the computer
- Overhead projector for the computer

Lesson 9 Procedure

A. Motivation and introduction

1. Introduce the students to the Geometer's Sketchpad™. Show them how to get on the computer and how to start Geometer's Sketchpad™.
2. Show students on the overhead projector how to draw and label points, undo a construction, start a new sketch. Have them construct several points.

3. Use the overhead projector to show students how to select points and construct line segments. Have the students do this on their computers.
4. Show students how to select the vertices of an angle and use Geometer's Sketchpad™ to measure the angle. Have the students do this on their computer.
5. Have the students create a triangle. Ask them to guess the sum of the interior angles of their triangle. Have them measure the three angles. "Will this (180^o) be the sum for every triangle? How could you test this conjecture?"
6. Show students how to select a vertex of the triangle and drag the vertex to create other triangles. Have them do this with their triangle. "What is the sum of the interior angles of this new triangle? Does your conjecture seem to hold?"
7. "Do you think the sum of the interior angles will stay the same, increase, or decrease if you create a polygon with more sides?" "How could you determine if you are correct?"
8. Suggest that the students work on this problem in teams.

B. Development (including discussion points and feedback)

1. Place the students in heterogeneous cooperative groups of about 4 students.
2. Assign each student a task. (leader, recorder, reporter, etc.)
3. Distribute the worksheet "The Sum of the Interior Angles of a Polygon."
4. Ask the teams to complete the investigation.
5. Circulate among the teams, guiding the students to complete the project, and observing the participation of students in the groups and their understanding of the mathematics concepts.
6. Ask questions to ensure that students know how to round off the sum of the interior angles to a whole number of degrees.
7. Make sure each team knows how to drag a vertex and that they know why showing that the conjecture holds for many different polygons of the same type implies that the conjecture is true, but is not a proof.
8. Help teams to decompose polygons into triangles. Guide them to discover a pattern to the number of triangles.
9. Guide students to find a relationship between the number of triangles in a polygon and the sum of the interior angles of that polygon.
10. Guide students to discover a rule for determining the sum of the interior angles of a polygon based on the number of its sides.
11. Assist students to see that a 20-sided polygon must have 20 interior angles. "In a 20-sided polygon, if each angle has the same measure, what must be the measure of each one, given the total (sum) of all the interior angles."
12. Ask each team to report on how to determine the sum of the interior angles of a particular polygon (quadrilateral, pentagon, etc.). Ask them to explain how they discovered the rule they are using. Encourage different approaches.

13. Ask questions to assess whether the students understand the concepts of vertex, decomposition, making and testing conjectures.
    - "What makes a point on a polygon a vertex?"
    - "If I add a triangle to a polygon, is that a decomposition? Why or why not?"
    - "How did you come up with your conjecture?"
    - "What makes you think your conjecture is reasonable?"
14. Ask all students if they could understand the team presenting their conjecture. That is, make sure all students are comfortable expressing the patterns in words.
15. Show equations for each pattern for a polygon with n sides. [For a polygon with n sides the number of triangles T is $T = n - 2$. For a polygon that can be decomposed into T triangles, the sum S of the interior angles is $S = 180\ T$ degrees. The sum of the interior angles of a polygon with n sides is $S = 180\ (n - 2)$].

C. Summary and closure

1. Ask teams to discuss and agree upon two things that learned (did) today.
2. Have teams report out and share what they learned. Guide responses to summarize the following:
    - The sum of the interior angles of a triangle is 180^o
    - Polygons can be decomposed into triangles
    - Any given polygon may be decomposed into n-2 triangle (where n is the number of sides of the polygon)
    - The sum(s) of the interior angles of a polygon of n sides is s=180 (n-2).
    - We learned how to navigate on the Geometer's Sketchpad^{Tm}

D. Assignment

Ask the students to determine the measure of each interior angle of a regular pentagon and a regular hexagon. If students are familiar with tilings of the plane, ask them, "Which of these regular polygons will tile the plane?" Ask them to justify their conclusions.

Assessment

- Observe the students during the investigation.
- Grade the group project, giving each team a group grade.
- Grade the homework, if you give them a rubric first.

Worksheet:

The Sum of the Interior Angles of a Polygon
A Geometer's Sketchpad™ Investigation

Start the Geometer's Sketchpad™ program. Click on "File|New Sketch."

> **To create points:** Click on the "Draw Point" icon. Draw three points for the vertices of the triangle.
>
> **To select points:** Click on the Select or Translate arrow icon. Click on the first point. While holding down the shift key, click on the other points. All should now be highlighted, with a circle around each point.
>
> **To draw line segments:** Make sure the vertices were selected in the order you want the line segments drawn. Click on "Construct|Segment."
>
> Create a triangle using Geometer's Sketchpad.

Measure each of the interior angles of the triangle you created.

- Select the 3 vertices so the second vertex is at the angle you want to measure.
- Click on "Measure|Angle"
- Repeat this for the other two angles.

Find the sum of the measures of the interior angles. Round the answer to the nearest degree. What is the sum of the interior angles of the triangle?

Select a vertex of the triangle and hold down the mouse button while moving the mouse in order to drag the vertex to a new location. Find the sum of the interior angles for this new triangle, rounded to the nearest degree.

Repeat this several more times. What did you discover? What is always the sum of the interior angles of any triangle? Why can you feel comfortable making this assertion?

Click on "File|Close|Don't Save."

Sum of the Interior Angles of a Quadrilateral (4-sided, Closed Figure)

Using Geometer's Sketchpad, draw four points in a clockwise direction for the four vertices of a quadrilateral. Select the four vertices in this order and construct the sides of the quadrilateral.
Find the measure of each interior angle as shown above.
Find the sum of the measures of the interior angles. Round the sum to the nearest degree.

Select a vertex of the quadrilateral and hold down the mouse button while moving the mouse in order to drag the vertex to a new location. Find the sum of the interior angles for this quadrilateral, rounded to the nearest degree.

Repeat this several more times. What did you discover?

In turn, select a different vertices and drag them to create new quadrilaterals. What is the sum of the interior angles for each of these new quadrilaterals?

What is always the sum of the interior angles of any quadrilateral? Why can you feel comfortable making this assertion?

Click on "File|Close|Don't Save."

Sum of the Interior Angles of a Pentagon (5-sided, closed figure)

Create a pentagon. Find the measures of each of the five interior angles. Find the sum of these measures, rounded to the nearest degree. What is the sum?

Select and drag a vertex of the pentagon to create a new pentagon. What is the sum of the interior angles for this pentagon?

Repeat this several times. What is the sum of the interior angles for any pentagon? Why can you feel comfortable making this assertion?

Look at the sum of the interior angles for a triangle, a quadrilateral, and a pentagon. Do you see a pattern to these sums? Describe the pattern.

Use the pattern to predict the sum of the interior angles of a hexagon (6-sided, closed figure).

Draw a hexagon and find the sum of the interior angles. Was your conjecture correct?

Click on "File|Close|Don't Save."

A More Formal Justification of Our Findings

We will attempt to find a rule for finding the sum of the interior angles of a polygon with a certain number of sides. Refer to the polygons shown below. Notice that each polygon can be divided into triangles by drawing straight lines from a single vertex to other vertices. Recall that the sum of the interior angles of a triangle is 180 degrees.

Complete the table below. Draw additional figures as needed.

# sides (*n*)	# triangles	sum of the interior angles
3	1	180 degrees
4	2	360 degrees
5		
6		
7		

Describe in your own words the pattern between the number of triangles that can be formed in this way and the number of sides of the original polygon.

If a polygon has 9 sides, how many triangles can be formed in this manner?

Describe in your own words the pattern between the number of triangles and the sum of the interior angles. That is, what must you do to the number of triangles in the convex polygon to obtain the sum of the interior angles answer in each instance?

If a polygon has 9 sides, what will be the sum of its interior angles?

Describe in your own words how you found the sum of the interior angles for a convex polygon with 9 sides.

Use the pattern you just found to find the sum of the interior angles of a 20-sided polygon.

What is the measure of each interior angle of a regular 20-sided polygon? (A **regular polygon** is a polygon with all sides **congruent** [same length] and all interior angles **congruent** [same measure]).
Explain how you found this answer